#### Mathematical model

ANNs are a subset of machine learning techniques, namely, supervised learning; in the present case ANN are used as a regression algorithm, since the outputs (RoCoF and nadir) are continuous variables. It is noteworthy that, however complex an ANN is, it is possible to extract certain parameters from a trained ANN (described below) such that it becomes possible to compute the outputs for a given set of inputs (OPs).



Figure 1 – ANN Architecture model (general example)

In this particular case, the architecture of the trained ANN is as follows: one hidden layer with 12 neurons. The activation function on the hidden layer is the sigmoid (described below) and the output layer has a linear function activation.

The inputs and outptus of the ANN are as follows:

* Inputs matrx, *[I]*, with dimension *i x 1*, where *i* is the number of inputs (features),
* Outputs matrix, *[O]*, with dimension *o x 1*, where *o* is the number of outputs (o=2, RoCoF and nadir in this case,

The parameters that need to be extracted from a trained ANN in order to compute the outputs as a function of inputs are the following:

* Weights matrix, *[WIH]*, with dimension *i* *x* *h*, where *h* is the number of neurons in the hidden layer,
* Bias array, *[BH]*, with dimension *h x* *1*,
* Weights matrix, *[WHO]*, with dimension *h x* *o*,
* Bias array, *[BO]*, with dimension *o x 1,*
* Maximum and minimum values for each explicative variable in the training set.

The steps for computing the outputs based on a set of inputs are as follows:

1. Transpose *[WIH]* (after which dimension becomes *h x i*),
2. Declare the input array *[I]*, with dimension *i x 1*, which contains the values of explicative variables
   1. Normalize each value *x* of the array using the expression, in relation to the maximum and minimum values for each explicative variable in the training set:
3. Obtain *[VH]*, the product of *[WIH]*.*[I]* which is an array with dimension *12 x 1*
4. Obtain *[H] = [VH] + [BH],* an array with dimension *12 x 1*
5. Obtain *[AH]* by running the sigmoid activation function *S(x)* on each element *x* of the array [*H*]:

1. Transpose *[WHO]* (after which dimension becomes *o x h* or *2 x 12*),
2. Obtain *[VO],* the product of *[WHO].[AH]*, which is an array with dimension *2 x 1*
3. Obtain *[O] = [VO] + [BO],* an array with dimension *2 x 1*
   1. Since a linear function is used on the output, there is no need to run any further activation function for each element of the array *[O]*
4. Denormalize the values of *[O]*